paper, applied by the method of longitudinal-transverse wrapping and utilizing the low vapor temperatures of the stored fluid in the temperature interval from 4.2 through 293 K.

#### NOTATION

 $Q_{ins}$ ,  $Q_{v.c}$ ,  $Q_{\Sigma}$ , heat flows: through the insulation, through the vapor-cooled throat section, total;  $\lambda$ , coefficient of thermal conductivity;  $(dT/dX)_{cold}$ , change in temperature at the cold end of the throat section or some segment of the latter; f, lateral cross-sectional area; F,  $F_h$ ,  $F_c$ , surface areas of the insulation: average, hot, and cold boundary surfaces;  $\delta$ , insulation thickness;  $T_h$ ,  $T_c$ , temperatures of the hot and cold ends of the throat sections;  $T_1$ ,  $T_2$ , boundary temperatures of the insulation layer; m, gas flow rate; r, heat of cryogenic-fluid vaporization;  $C_p$ , heat capacity of the gas. Subscripts: ins, insulation; w, wall; pl, plug; g, gas; av.eff, average effective; i, number of throat segments and insulation; b, boiling.

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# EFFECT OF INJECTING AN ION BEAM IN A DENSE PLASMA ON THE STRUCTURE OF THE PERTURBED ZONE

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A hydrodynamic description of a weakly ionized dense plasma with constant properties and frozenin reactions of formation and the extinction of charged particles serves as the basis for our examination of the influence exerted by the injection of a beam of negative ions on the characteristics of the wall layer.

The study of the interaction of beams of charged particles with a plasma is one of the primary problems confronting plasma physics. The substantial difficulties which arise in the utilization of electron beams, as well as the considerable achievements attained in the development of powerful apparatus for the pulsed generation of strong ion beams, has led to a situation in which greater attention is being devoted to the latter.

We examine the plasma-beam formation restricted by a nonconducting wall into which has been built an electrode whose potential may vary relative to the surrounding plasma. The configuration of the beam-plasma system is shown in Fig. 1.

Let us examine an axisymmetric ion beam propagated through a plasma, parallel to an external magnetic field  $B_0e_z$ . If the axial current of the plasma is only partially neutralized by a countercurrent flowing through the plasma, the resulting axial current will generate an intrinsic azimuthal magnetic field  $B_{\theta}^{s}(r, z)e_{\theta}$ . If, in addition, the space charge of the beam is only partially neutralized by the surrounding plasma, while the electrode is subjected to some potential, then the deviation of the charge from a neutral state will lead to the appearance of radial  $E_r$  and axial  $E_z$  electric fields.

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Fig. 1. Plasma beam column: 1) wall; 2) electrode; 3) beam.

For purposes of analyzing the system, let us make a number of simplifying assumptions: 1) the electrode is shaped in the form of a disk whose axis is oriented parallel to the external magnetic field; 2) the external axial magnetic field  $B_0e_z$  is uniform; 3) the distribution of all physical quantities is azimuthally uniform relative to the axis of the magnetic field  $B_0$ ; 4) the azimuthal current  $j_{\theta}(\mathbf{r}, t) = \sum_{\alpha} q_{\alpha} n_{\alpha}(\mathbf{r}, t) V_{\theta\alpha}(\mathbf{r}, t)$ , which generates the axial magnetic field  $B_z^{s}(\mathbf{r}, z)e_z$ , is rather small, so that the intrinsic magnetic field can be treated as negligibly small in comparison to the external field, i.e.,  $|\mathbf{B}_z^{s}| \ll |\mathbf{B}_0|$ ; 5) up to the instant of time  $t_0$ : a) the external field  $\mathbf{B}_0$  increases according to some law; b) there is no beam injection; c) we neglect the intrinsic magnetic fields; for the times  $t > t_0$ : a) the external magnetic field is constant; b) a nonrelativistic ion beam is injected.

In the case of a uniform external magnetic field  $B_0(t)$  the strength of the electric field  $E_{\theta}$  will be determined from the expression:

$$E_{\theta}(r, t) = -\frac{r}{2c} \frac{dB_0}{dt} . \tag{1}$$

(4)

Within the scope of the adopted assumptions, we have the following significant field components:

$$\mathbf{E}(r, \ \theta, \ z, \ t) = \{E_r(r, \ z, \ t), \ E_{\theta}(r, \ t), \ E_z(r, \ z, \ t)\};$$

$$\mathbf{B}(r, \ \theta, \ z, \ t) = \{0, \ B_{\theta}^{s}(r, \ z, \ t), \ B_{\theta}(t)\},$$
(2)

where the components  $E_r$ ,  $E_z$ , and  $B_{\theta}^{s}$  are determined from the self-consistent Maxwell equations:

$$\frac{\partial B_{\theta}^{s}}{\partial z} = -\frac{1}{c} \frac{\partial E_{r}}{\partial t} - \frac{4\pi}{c} j_{r}; \quad \frac{\partial E_{r}}{\partial z} - \frac{\partial E_{z}}{\partial r} = -\frac{1}{c} \frac{\partial B_{\theta}^{s}}{\partial t};$$

$$\frac{1}{r} \frac{\partial}{\partial r} r B_{\theta}^{s} = \frac{1}{c} \frac{\partial E_{z}}{\partial t} + \frac{4\pi}{c} j_{z}; \quad \frac{1}{r} \frac{\partial}{\partial r} r E_{r} + \frac{\partial E_{z}}{\partial z} = 4\pi\rho.$$
(3)

For electromagnetic fields (2) the equations of the macroscopic model in the case of thermodynamic equilibrium can be written in the form

$$\begin{split} \frac{\partial n_{\alpha}}{\partial t} &+ \frac{1}{r} \frac{\partial}{\partial r} \left\{ \frac{1}{1 + \Omega_{\theta_{\alpha}}^{2} + \Omega_{z_{\alpha}}^{2}} r \left[ \left( \mu_{\alpha} n_{\alpha} E_{r} - D_{\alpha} \frac{\partial n_{\alpha}}{\partial r} \right) + \right. \\ &+ \Omega_{z_{\alpha}} \mu_{\alpha} n_{\alpha} E_{\theta} - \Omega_{\theta_{\alpha}} \left( \mu_{\alpha} n_{\alpha} E_{z} - D_{\alpha} \frac{\partial n_{\alpha}}{\partial z} \right) \right] \right\} + \\ &+ \frac{\partial}{\partial z} \left\{ \frac{1}{1 + \Omega_{\theta_{\alpha}}^{2} + \Omega_{z_{\alpha}}^{2}} \left[ \Omega_{\theta_{\alpha}} \left( \mu_{\alpha} n_{\alpha} E_{r} - D_{\alpha} \frac{\partial n_{\alpha}}{\partial r} \right) + \right. \\ &+ \Omega_{\theta_{\alpha}} \Omega_{z_{\alpha}} \mu_{\alpha} n_{\alpha} E_{\theta} + (1 + \Omega_{z_{\alpha}}^{2}) \left( \mu_{\alpha} n_{\alpha} E_{z} - D_{\alpha} \frac{\partial n_{\alpha}}{\partial z} \right) \right] \right\} = 0, \end{split}$$



Fig. 2. Evolving curves for charged particles flowing to the electrode when  $\varphi_0$  < 0: 1)  $n_p^0 = 0.5$ ,  $U_p^0 = 2$ ; 2)  $n_p^0 = 0.5$ ,  $U_p^0 = 3$ .

where

$$\Omega_{\theta_{\alpha}} = \frac{q_{\alpha} B_{\theta}^{s}}{m_{\alpha} v_{\alpha} c} ; \ \Omega_{z_{\alpha}} = \frac{q_{\alpha} B_{0}}{m_{\alpha} v_{\alpha} c} ; \ D_{\alpha} = \frac{k T_{\alpha}}{m_{\alpha} v_{\alpha}} ; \ \mu_{\alpha} = \frac{q_{\alpha}}{m_{\alpha} v_{\alpha}}$$

The remaining notation is analogous to [1]. The relationship between the systems of equations (3) and (4) is self-evident.

Up until the instant of injection onset, the initial and boundary conditions for system of equations (4) can be formulated in a manner identical, for example, to [1, 2]. The resulting steady solution of problem (4) serves as the initial conditions in the beam injection stage.

As a consequence of the collision of the beam particles with the plasma components the beam velocity  $U_p$  after its injection into the plasma will relax over time according to the equation

$$m_p n_p \left(\frac{\partial}{\partial t} + \mathbf{U}_p \nabla\right) \mathbf{U}_p = q_p n_p \left\{ \mathbf{E} + \frac{1}{c} \left[ \mathbf{U}_p, \mathbf{B} \right] \right\} - \nabla k T_p n_p - m_p n_p \mathbf{U}_p \mathbf{v}_p, \tag{5}$$

where the subscript p pertains to the injected particles, while the beam ion collision frequency  $\nu_p$  for the weakly ionized gas can be determined in accordance with the theory developed in [3].

The boundary conditions for the plasma are conserved at the instant of time  $t_0$ , corresponding to the onset of injection. The velocity of the negative ions in the injection plane  $U_p(r \in G, t) = U_p^0(t)$ , where  $U_p^0(t)$  represents the unperturbed beam velocity and G defines the surface of the electrode. At the outside boundary, in order to achieve smoothness, we can make use of a linear, quadratic, or cubic extrapolation, as is done for fluid flows [4].

The equation of motion (5) for beam particles must be completed with the continuity equation

$$\frac{\partial n_p}{\partial t} + \operatorname{div} n_p \mathbf{U}_p = 0 \tag{6}$$

having the conditions

$$n_p(\mathbf{r}, t_0) = 0; \ n_p(\mathbf{r} \in G, t) = n_p^0; \ n_p(\mathbf{r}_{\infty}, t) = 0,$$
 (7)

where  $n_p^0$  is the density of the beam at the axis of system symmetry for the case in which z = 0.

The problem in such a formulation permits of no analytical solution. With numerical modeling to calculate electromagnetic fields we made use of the algorithm proposed in [5], which allows us to carry out our calculations with



Fig. 3. Profiles for the distribution of charged-particle concentrations; notation the same as in Fig. 2.





grid dimensions considerably in excess of the Debye length. With this method it is possible to minimize the numerical diffusion and, moreover, it is capable of suppressing the instabilities associated with plasma waves. For solution of the continuity equations we employed the large-particle method [6], for which the system of equations (4) was brought to divergent form. A uniform grid with a dynamic outer boundary was used in the calculations, and this can be ascribed to the space-time development of the beam. The problem was solved in dimensionless form [1].

Figures 2-4 show some of the computational results with respect to the cited model for beams of varying intensities, given linear electrode dimensions of  $r_0 = 5$  and surface potentials of  $\varphi_0 = \pm 10$ .

For electrodes with negative displacement at the start of the evolution of the perturbed zone, the presence of the beam leads to a reduction in the flow of positive plasma ions to its surface, which is a consequence of the formation at the surface of an uncompensated negative space charge and, consequently, of a potential hole. With an increase in the velocity at which the beam is injected into the plasma, given identical particle density through the cross section, the ion flux attains a steady value more slowly. The steady-state value of the ion current to the electrode is lower than its value for the case in which  $j_p^{0} = 0$ . Moreover, with an increase in the velocity  $U_p^{0}$  of beam injection in the evolution ion-current curves the appearance of pulsations becomes possible (Fig. 2). With negative values of  $\varphi_0$  the electron conductivity current  $j_e$  to the surface of the electrode, exhibiting a relatively high value at the start of the evolution process, rapidly diminishes, experiencing no oscillations. Some increase in the value of  $j_e$  initially, in the presence of the beam, above its value for the case in which  $j_p^{0} = 0$  is caused by the change in the sign of the electric-field strength vector in the immediate vicinity of the injected surface. With the development of the processes over time the potential hole is dispersed and the electron current diminishes all the more rapidly, the more intensive the beam (Fig. 2).

With variation of  $\mathbf{j}_p^0$  the electron curves  $n_e(z)$  substantially "bend toward" the coordinate axes (Fig. 3). In the presence of the beam, if the magnetic field  $\mathbf{B}_0$  in the z direction does not affect the behavior of the curves, then in the radial direction the curves  $n_e(r)$  move closer to each other. In the  $n_i(r)$  profiles over the entire series of calculations with  $\varphi_0 < 0$  at the imaginary boundary of the imagined cylinder of radius  $r_0$  we observed an additional minimum whose appearance can be explained by the structure of the electromagnetic field.

The calculations carried out for positive values of the potential  $\varphi_0$  demonstrated some divergence between the evolution and stationary characteristics relative to the case in which  $\varphi_0 < 0$  (Fig. 4). The relaxation curves  $j_i(t)$  have values

that are somewhat too low in order to introduce any correction factors into the resulting current to the surface of the electrode, and the time over which substantial values of  $j_i$  were observed are reduced as the magnetic field  $B_0$  is intensified and as the intensity of the beam is increased. The evolution curves for the electron current are significantly distorted only during the initial period. Subsequently,  $j_e(t)$  is stabilized, tending to values prevailing in the absence of injection. Some increase in  $j_e$  for small t is caused by the formation of potential holes and increasing potential gradients near the surface. Moreover, in the curves  $j_i(t)$  and  $j_e(t)$  we observed no oscillations characteristic for the case  $\varphi_0 < 0$ . The reason for this should apparently be sought in the increase in the focusing forces. With positive  $\varphi_0$  the increase in  $j_p^0$  may lead to the appearance of a flow of negative ions in the direction of the injected surface. Calculations have shown that the values of  $j_p$  at the electrode diminish with an increase in  $U_p^0$ , given equal beam densities, and they increase with an increase in  $n_p^0$  for equal  $U_p^0$ . The behavior of  $n_\alpha(r, z)$  with positive potentials qualitatively does not differ from the case  $\varphi_0 < 0$ .

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